Midterm 1 – Review – Problems

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Problem 1:

(1.9) Prove that the union of two subspaces of V is a subspace of V if and only if one of them is contained in the other.

Problem 2:

Find a vector space V and three subspaces U_1, U_2, U_3 of V such that:

(1) $V = U_1 + U_2 + U_3$

(2) $U_1 \cap U_2 = \{\mathbf{0}\}$, $U_1 \cap U_3 = \{\mathbf{0}\}$, $U_2 \cap U_3 = \{\mathbf{0}\}$

(3) **BUT** $V \neq U_1 \oplus U_2 \oplus U_3$

Hint: Let $V = \mathbb{R}^3$. If you *really* want to see the solution, it's on page 16 of the book!

Problem 3:

(2.10) Suppose $\dim(V) = n$. Prove that there are $1-$dimensional subspaces $U_1, \cdots, U_n$ of V such that $V = U_1 \oplus \cdots \oplus U_n$

Problem 4:

((2.17), with $m = 2$) Suppose V is finite-dimensional and $V = U \oplus W$. Show that $\dim(V) = \dim(U) + \dim(W)$

Problem 5:

Prove or give a counterexample: If U_1, U_2, U_3 are subspaces of V, then:

$$(U_1 + U_2) \cap U_3 = (U_1 \cap U_3) + (U_2 \cap U_3)$$